Multiple Description Image Coding

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Presentation Layout

- Project objectives
- MD (Multiple Description) – motivation
- System overview
- System implementation
- Results
- Possible improvements
- Conclusions
Project Objectives

- Understanding the concepts of MD coding
- Building an MD coding system for images
- Performance and robustness analysis of the system
Motivation

Where does an SD (Single Description) coding system go wrong?

- Packet losses!
  - Intolerable retransmission delay
  - No feedback channel
  - Order must be maintained (layered coding)
Motivation (cont.)

Solution:

- Add correlation between packets and estimate the lost packets

⇒ Make all received packets useful
Introduction to MD

Example:

Layered:
- All 4 packets received

Non-layered:
- Packet #3 is lost

Layered:
- "x" symbol

MD:
- Original image

29/11/05
Applications

- Applications - lossy environments:
  - Audio
  - Images
  - Video
- Packet networks
  - Example: Internet
System Layout

Original image → Apply block-wise DCT transform → Quantize the DCT coefficients (JPEG) → Pair the quantized DCT coefficients → Apply a correlating transform → A correlated pair is created → Decode according to what is received → Channel 1 → Decoder 1 → Channel 2 → Decoder 2

Decoder 0
Pairing Coefficients

- Choose maximally uncorrelated pairs

Quantized DCT blocks

Pair high-variance coeff. with low-variance coeff.
DCT Block Statistics

- Variances were obtained from a large set of images
- Means are assumed to be zero (except DC coef.)
System Implementation

Correlating Transform
MD Correlating Transform

Add correlation to create the output pair $Y$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$
System Implementation

Decoding

29/11/05

Multiple description image coding
Decoding

Central decoder - full reconstruction

\[
\begin{bmatrix}
    d & -b \\
    -c & a
\end{bmatrix}
\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix}
= 
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
\]
Decoding (cont.)

Side decoder 1 - partial reconstruction

\[ \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \]

Optimal estimation of \( y_2 \)

\( \hat{y}_2 \) was estimated

\( \hat{x}_1 \)
System Implementation

Decoder 1
Decoder 0
Decoder 2
General Assumptions

- Paired coefficients are independent Gaussian random variables

- Both channels have equal failure probability
Correlating Transform
Correlating Transform (cont.)

For balanced descriptions: 
\[ r_1 = r_2 = \frac{1}{\sqrt{\sin 2\theta_1}} \], \quad \theta_2 = -\theta_1

Finally:
\[ T = \begin{bmatrix} \sqrt{\cot \theta_1} & \sqrt{\tan \theta_1} \\ \frac{\cot \theta_1}{2} & \frac{\tan \theta_1}{2} \\ -\sqrt{\cot \theta_1} & \sqrt{\tan \theta_1} \end{bmatrix} \]
Redundancy Allocation

\[
R_{x_1x_2} = \begin{bmatrix}
\sigma_x^2 & 0 \\
0 & \sigma_y^2
\end{bmatrix} \implies R_{y_1y_2} = \begin{bmatrix}
\sigma_x^2 & \sigma_x \sigma_y \cos \phi \\
\sigma_x \sigma_y \cos \phi & \sigma_y^2
\end{bmatrix}
\]

Let \( R^* \) be the rate of \( x_1 \) & \( x_2 \) at \( D_0 \)
Let \( R \) be the rate of \( y_1 \) & \( y_2 \) at \( D_0 \)

\[
\Rightarrow \rho = R - R^* = -\frac{1}{2} \log_2 \sin \phi \ [\text{bpp}]
\]

RRD (Redundancy Rate Distortion) function
Redundancy Allocation (cont.)

For optimal redundancy allocation, each MD coder should operate at the same slope on its RRD curve.

\[ \sum_{m=1}^{M} \rho_m = \frac{1}{M} \]

\[ \lambda = -\frac{\partial D_{1,m}(\rho_m)}{\partial \rho_m} \]

Flow:

- Choosing redundancy \( \rho \) yields \( \lambda \), and in turn the different \( \rho_m \)'s
Correlating Transform

- Calculate different transformation for each pair

\[
R_{y_1y_2} = \begin{bmatrix}
\sigma^2_{y_1} & \sigma_{y_1}\sigma_{y_2}\cos\phi_m \\
\sigma_{y_1}\sigma_{y_2}\cos\phi_m & \sigma^2_{y_2}
\end{bmatrix}
\]

\[
\tan\theta_m = \frac{\sigma_{x_{1,m}}}{\sigma_{x_{2,m}}} \tan\frac{\phi_m}{2}
\]

\[
T_m = \begin{bmatrix}
\sqrt{\frac{\cot\theta_{1,m}}{2}} & \sqrt{\tan\theta_{1,m}} \\
-\sqrt{\frac{\cot\theta_{1,m}}{2}} & \sqrt{\tan\theta_{1,m}}
\end{bmatrix}
\]
Correlation Factor

Correlation coefficient vs Redundancy ($\rho$) [bpp]

- Pair 1 (2&64)
- Pair 7 (8&58)
- Pair 15 (16&50)
Arithmetic coders are used

- Each transform coefficient has different statistics and is coded separately. Initial statistics are given to each coder
- Better performance for larger images
DC Coding

- DC Coefficients:
  - Takes most of the redundancy
  - Can be coded quite efficiently (DPCM)
  - Sent on both descriptions
Results - Rate

\[ R(\rho) = R^* + \rho \]
Decision Making

Number of coefficients to send

PSNR₁ [dB] vs Num of coef. sent

Rate (R) [bpp] vs Num of coef. sent
Results – Packet Lost, No Redundancy

Only the upper part is received

\[ R^* = 0.7 \text{ [bpp]} \]

\[ \text{PSNR}_1 = 25 \text{ [dB]} \]

(Without sending DC twice)
Results – Packet Lost, Minimal Redundancy

\[ R^* = 0.7 \text{ [bpp]} \]
\[ \text{PSNR}_1 = 29.9 \text{ [dB]} \]
\[ \text{Redundancy} = 0.1\% \]
\[ (\rho \approx 10^{-3} \text{ [bpp]} \) \]
Results – Packet Lost, Small Redundancy

\[ R^* = 0.7 \text{ [bpp]} \]
\[ \text{PSNR}_1 = 32.1 \text{ [dB]} \]
\[ \text{Redundancy}=15\% \]
\[ (\rho = 0.1 \text{ [bpp]}) \]
Results – RRD Performance when a packet is lost

$R^* = 0.7 \text{ [bpp]}$

$\text{PSNR}_0 = 35.5 \text{ [dB]} \quad (R_{JPEG}=0.63 \text{ [bpp]})$
Possible Improvements

- Better partitioning of the image
  - Define blocks with common statistics, i.e. smooth, vertical edges etc.
- Use another transform instead of DCT
- Create more than two descriptions
- Can be extended to be optimal for non-symmetric channels
Conclusions

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THE END